Math 1050 Exam 5 Review

1. $\dfrac{7!}{4!} = \dfrac{7\cdot 6\cdot 5\cdot 4!}{4!} = 7\cdot 6\cdot 5 = \boxed{210}$

2. a) $s_n = 4n-2$

$s_1 = 4(1)-2 = 2$

$s_2 = 4(2)-2 = 6$

$s_3 = 4(3)-2 = 10$

$s_4 = 4(4)-2 = 14$

$s_5 = 4(5)-2 = 18$

$2, 6, 10, 14, 18$

b) $a_n = 2n^2+n$

$a_1 = 2(1)^2+1 = 3$

$a_2 = 2(2)^2+2 = 10$

$a_3 = 2(3)^2+3 = 21$

$a_4 = 2(4)^2+4 = 36$

$a_5 = 2(5)^2+5 = 55$

$3, 10, 21, 36, 55$

3. $a_1 = 8, \quad a_n = 5a_{n-1}+2$

$a_1 = 8$

$a_2 = 5(8)+2 = 42$

$a_3 = 5(42)+2 = 212$

$a_4 = 5(212)+2 = 1062$

$8, 42, 212, 1062$

4. $\displaystyle\sum_{k=1}^{n} (3k-1) = 2+5+8+11+\cdots+(3n-1)$

5. $4^3 + 5^3 + 6^3 + \ldots + 13^3 = \boxed{\sum_{k=4}^{13} k^3}$

6. $\sum_{k=2}^{5} (3k+7) = 13 + 16 + 19 + 22 = \boxed{70}$

or arithmetic sequence: $S = \frac{n}{2}(a_1 + a_n)$

$S = \frac{4}{2}(13+22) = \boxed{70}$

7. $a_1 = 5$, $d = 3$

$a_n = 5 + (n-1)(3) = 5 + 3n - 3 = 3n + 2$

$$\boxed{\begin{aligned} a_n &= 3n+2 \\ a_{18} &= 3(18)+2 = 56 \end{aligned}}$$

8. $n = 28$, $a_1 = 18$, $d = 2$

$S = \frac{n}{2}(a_1 + a_n)$

$a_n = 18 + (28-1)(2) = 72$

$S = \frac{28}{2}(18 + 72) = \boxed{1260}$

9. $a_7 = -47$ $\quad a_{13} = -101$

use: $a_n = a_1 + (n-1)d$, to find a_1 & d.

$-47 = a_1 + (7-1)d$ | $-101 = a_1 + (13-1)d$

$-47 = a_1 + 6d$ | $-101 = a_1 + 12d$

Eliminate a_1

$$\begin{aligned} -47 &= a_1 + 6d \\ -(-101 &= a_1 + 12d) \\ \hline 54 &= -6d \\ -9 &= d \end{aligned}$$

solve for a_1 using $d = -9$

$-47 = a_1 + 6(-9)$

$7 = a_1$

First term: $a_1 = 7$

Common diff.: $d = -9$

Recursive rule: $a_1 = 7$, $a_n = a_{n-1} - 9$

10. $(-5) + (-2) + 1 + 4 + \cdots + 76$

arithmetic sequence: $\text{sum} = \frac{n}{2}(a_1 + a_n)$

$d_1 = 3,\ a_1 = -5$, $a_n = 76$, Find n

$$76 = -5 + (n-1)(3)$$

$$76 = -5 + 3n - 3$$

$$76 = 3n - 8$$

$$84 = 3n$$

$$28 = n$$

$$S = \frac{28}{2}(-5 + 76)$$

$$= 14(71) = \boxed{994}$$

11. $-1, \frac{1}{2}, -\frac{1}{4}, \ldots$

$a_n = a_1 \cdot r^{n-1} \Rightarrow a_n = -1\left(-\frac{1}{2}\right)^{n-1}$

$a_{10} = -1 \cdot \left(-\frac{1}{2}\right)^9 = \boxed{\frac{1}{512}}$

12. $a = 3,\ r = 4$

$\boxed{a_n = 3 \cdot (4)^{n-1} \qquad a_6 = 3 \cdot (4)^5 = 3072}$

13. $a = 2$, $r = 3$

$\boxed{a_n = 2(3)^{n-1}}$

14. $\sum_{k=1}^{5} \left(\frac{1}{2}\right)(2)^k$

$S = a_1\left(\frac{1-r^n}{1-r}\right)$

$S = 1\left(\frac{1-2^5}{1-2}\right) = \boxed{31}$

15. a) diverges, $r=2$

b) converges, $r=\frac{-1}{2}$ $\quad S=\frac{a}{1-r}$ $\quad \frac{4}{1-(\frac{-1}{2})}=\frac{8}{3}$

16. a) $-5n+2 \rightarrow$ linear so, arithmetic
$d=-5$

b) $4n^2+7 \rightarrow$ neither (quadratic)

c) 3^{2n} exponential so, geometric $\quad r=3^2$

17. $\binom{12}{3} = \frac{12!}{3!\,9!} = \frac{\overset{2}{\cancel{12}}\cdot 11\cdot 10\cdot \cancel{9!}}{\cancel{3!}\cdot \cancel{9!}} = \boxed{220}$

18. $\binom{8}{2} = \frac{8!}{2!\,6!} = \frac{\overset{4}{\cancel{8}}\cdot 7\cdot \cancel{6!}}{\cancel{2}\cdot \cancel{6!}} = \boxed{28}$

19. $(x+1)^5$

$(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

$a=x$
$b=1$ $\quad \boxed{(x+1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1}$

20. $\binom{5}{1}(4x)(6)^4 = 5\cdot 4\cdot 1296 = \boxed{25{,}920}$

21. $\binom{4}{2}(5x)^2(3)^2 = 6\cdot 25x^2\cdot 9 = \boxed{1350x^2}$